Sec. 14.5:
The Chain Rule (Calc. III version)

What We Will Go Over In Section 14.5

1. The Chain Rule (Calc. III version)
2. Implicit Differentiation

1. The Chain Rule

Calc 1 Version…

Recall: The Chain Rule is the rule you use to find the derivative of a composition of functions

Ex: Find…

$$\frac{d}{dx}\sin x \qquad\qquad \frac{d}{dx}\sin(e^x) \qquad\qquad \frac{d}{dx}\sin(u)$$

If you call the "stuff" u , then the Calc. I chain rule can be written as:

$$\frac{df}{dx} = \frac{df}{du}\cdot\frac{du}{dx} \quad \text{or} \quad \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$\begin{tikzcd} y \text{ (or } f \text{)} \arrow[d, dash] \\ u \arrow[d, dash] \\ x \end{tikzcd}$$

1. The Chain Rule

Calc 3 Version…

- There are many "stuffs"
- Think of f as a function of many variables (not independent) and that each of these variables are functions of other variables

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So if f (or z) is a function of x and y, and each of x and y are functions of r, s and t, then…

<u>Tree</u>

<u>Chain Rule</u>

1. The Chain Rule

Ex 1:

Example 1

If $z = x^2y + 3xy^4$, where $x = \sin 2t$ and $y = \cos t$, find dz/dt when $t = 0$.

1. The Chain Rule

Ex 2:

Example 3

If $z = e^x \sin y$, where $x = st^2$ and $y = s^2 t$, find $\partial z/\partial s$ and $\partial z/\partial t$.

1. The Chain Rule

Ex 3:

Example 4

Write out the Chain Rule for the case where $w = f(x, y, z, t)$ and

$x = x(u, v)$, $y = y(u, v)$, $z = z(u, v)$, and $t = t(u, v)$.

1. The Chain Rule

Ex 4:

Example 5

If $u = x^4 y + y^2 z^3$, where $x = rse^t$, $y = rs^2 e^{-t}$, and $z = r^2 s \sin t$, find the value of $\partial u / \partial s$ when $r = 2$, $s = 1$, $t = 0$.

1. The Chain Rule

Ex 5:

Example 7

If $z = f(x, y)$ has continuous second-order partial derivatives and $x = r^2 + s^2$ and $y = 2rs$, find (a) $\partial z/\partial r$ and (b) $\partial^2 z/\partial r^2$.

2. Implicit Differentiation

Implicit Differentiation (Calc. I situation)

If $F(x, y) = k$ defines y implicitly as a function of x, then

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$$\frac{dy}{dx} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} = -\frac{F_x}{F_y}$$

2. Implicit Differentiation

Ex 6:

Example 8

Find y' if $x^3 + y^3 = 6xy$.

2. Implicit Differentiation

Implicit Differentiation (Calc. III situation)

If $F(x, y, z) = k$ defines z implicitly as a function of x and y, then

$$\boxed{7} \qquad \frac{\partial z}{\partial x} = -\frac{\dfrac{\partial F}{\partial x}}{\dfrac{\partial F}{\partial z}} \qquad \frac{\partial z}{\partial y} = -\frac{\dfrac{\partial F}{\partial y}}{\dfrac{\partial F}{\partial z}}$$

2. Implicit Differentiation

Ex 7:

Example 9

Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $x^3 + y^3 + z^3 + 6xyz = 1$.